145AA Advanced Mechanics (Meccanica Superiore) Second semester 2025-2026 6 CFU, 42 hours

Instructors:

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Overview

The aim of the course is to give an introduction to the study of chaotic dynamical systems, both from the qualitative and the quantitative point of view. The course "Sistemi dinamici" provides an introduction to the notion of chaotic system from the point of view of the topological properties of a system. The two courses "Ergodic Theory" and "Mathematical Physics" focus, respectively, on ergodic theorems and (near) integrable systems. This course aims to study the the statistical properties of (close to) hyperbolic dynamical systems, through the lenses of the suitable operators.

In the first part of the course, we quickly introduce a minimum of ergodic theory i.e. how measure theory helps us understand the behaviour of a dynamical system. Next, we introduce some examples of chaotic maps and introduce invariants which allow to characterize them, such as metric entropy and Lyapunov exponents. Last, the quantitative study of such systems is explored in depth by means of transfer operators: we will see how we can read the rate of decay of correlations, the central limit theorem and large deviation theorem from the spectral properties of our system. According to the needs and likings of the participants, more geometric or more analytical arguments will be used, allowing to explore a variety of topics such as distribution of orbits via zeta functions, renormalization methods or perturbative methods.

Course outline

- Ergodic Theory 101.
- Invariants of dynamical systems: topological entropy; metric entropy; Lyapunov exponents.
- Transfer operators and the spectral approach.
- Mixing and decay of correlations. Further statistical properties.

Bibliography

- V. Baladi, "Positive transfer operators and decay of correlations", World Scientific, 2000
- M. Brin, G. Stuck, "Introduction to dynamical systems", Cambridge University Press, 2002

Assessment

The exam is oral. Students who attended the course may decide to give a seminar on an advanced topic previously agreed with the instructor. A list of topics will be provided during the course. The seminar is followed by a short discussion on the main results of the course.

Prerequisites

Basic notions of dynamical systems, measure theory and functional analysis.