Category theory

Instructors: Andrea Di Lorenzo, Mattia Talpo

Introduction

In mathematics there are many instances of the following pattern: one introduces some kind of structure of interest (say, a vector space), and then a notion of morphism between such structures - typically (though not always!) a function between the underlying sets that respects the structure (for example, a linear map).

A category is an abstraction of this pattern: it consists of specified collections of objects and transformations, called morphisms, between them. Groups, modules, topological spaces, measure spaces, ordinals, and so forth form categories. The action of packaging each variety of objects into a category shifts one's perspective from the particularities of each mathematical sub-discipline to potential commonalities between them. Abstract results in category theory can typically be specified to several different particular contexts, sometimes highlighting new connections between different areas. In particular, this language is fundamental in certain parts of algebra and geometry, but is also sometimes useful in other contexts.

The course will present the abstract theory - sometimes reputed to be rather dry - but will also contain plenty of examples taken from the rest of the mathematical curriculum, especially of the first two years. It is therefore primarily aimed at third-year students and beyond, but determined second-year students may also find it accessible.

Topics

In the first part of the course we will cover the basics of the language of category theory, such as functors, limits, adjunctions, etc. For each of these concepts, we will provide many examples, mostly (but not only) drawn from the areas of algebra, geometry and topology; the goal of this top-to-bottom approach will be to showcase the usefulness of the categorical language and its scope of application.

The second part will focus on more advanced topics. These will be determined later, depending also on the input from the students attending the course.

Practical information

This is a 42 hours course, and will take place in the second semester. The language (English or Italian) will be decided together with the interested students at the start of the course.

The exam will consist of a 30-minute presentation on an advanced topic (to be decided with the instructors), followed by some questions about the material covered in the course.

Prerequisites are almost non-existent for the general theory, but the examples will draw from material from the courses of the first two years.

References

[1] E. Riehl, Category theory in Context. Dover Publications, 2017.

[2] S. MacLane, Categories for the Working Mathematician. Springer-Verlag, New York, (1971) Graduate Texts in Mathematics, Vol. 5.