Mathematical Methods of Quantum Mechanics

Master Course in Mathematics @ Unipi,
2025-2026, Second Semester
6 CFU

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Course description

Quantum mechanics is one of the central theories of physics, whose language describes a huge variety of systems, from elementary particles to superconducting materials. Fascinating mathematical problems arise in this context, posing hard challenges in functional analysis, partial differential equations and operator theory. This course is an introduction to the rigorous mathematical framework of quantum mechanics and functional analytic methods for the study of the Schrödinger equation.

The course consists of two parts. The first part (of approximately 25 hours) will start with an overview on the theory of (unbounded) linear operators on Hilbert spaces and self-adjointness. We will then study properties of Schrödinger operators and quantum dynamics. Some selected topics related to the physical motivations of quantum mechanics will be presented.

The second part (of approximately 15 hours) will be devoted to many-body quantum mechanics. We will introduce the mathematical questions and tools in this context and conclude with a discussion of current research topics.

Class requirements

No previous knowledge of quantum mechanics is expected, but a background in analysis will be useful. Some concepts in functional analysis will be recalled along the way.

Exam Information

Written and oral exam.

Course Outline

First part: the mathematical framework of quantum mechanics

1. The formalism and motivation of quantum mechanics.
2. Theory of unbounded operators and self-adjointness.
3. Perturbations to the Laplacian: Kato-Rellich theorem, Schrödinger operators.
4. Schrödinger equation, strongly continuous unitary groups and generators. Quantum Noether theorem.

Second part: many-body quantum mechanics.

1. Trace-class and Hilbert-Schmidt operators. Tensor product of Hilbert spaces.
2. Interacting Hamiltonians and second quantization.
3. Derivation of the Hartree equation. Bose-Einstein condensation, Bogoliubov theory.

References

Main references

- Gerald Teschl: Mathematical Methods in Quantum Mechanics, With Applications to Schrödinger Operators, American Mathematical Soc. (2009)
  `https://www.mat.univie.ac.at/~gerald/ftp/book-schroe/index.html`
- Michael Reed, Barry Simon: Methods of Modern Mathematical Physics, Academic Press, Volume I-IV (1970)
- Elliott H. Lieb, Michael Loss: Analysis, American Mathematical Soc. (2001)

Other references

- Stephen J. Gustafson, Israel Michael Sigal, Mathematical Concepts of Quantum Mechanics.
  `https://www.math.utoronto.ca/~sigal/semlectnotes/1.pdf`
- I. Rodnianski, B. Schlein. Quantum fluctuations and rate of convergence towards mean-field dynamics. Comm. Math. Phys. 291 (2009)
  `https://arxiv.org/abs/0711.3087`

Class Material

Notes will be uploaded on the Teams page of the course.