Algebraic Number Theory 1
(TAN1)

A.A. 2025/26 - Second semester

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Introduction

The course illustrates the basic results in the theory of number fields, i.e., finite extensions of the rational field $\mathbb{Q}$. The course focuses on the main properties of their ring of integers (the integral closure of the ring $\mathbb{Z}$): factorization of ideals, structure of units and class groups.

The topics presented here (together with the theory of local fields studied in the course Algebraic Number Theory 2) provide the theoretical foundations for research in all areas of Algebraic Number Theory and (to some extent) Arithmetic Geometry.

Main Features

As a first introduction to number fields, the course focuses on the main properties of their ring of integers. We study their structure as Dedekind domains and focus on the unique factorization of ideals in (finite) products of powers of prime ideals. Such factorization is described via invariants (ramification indices and inertia degrees) which we shall study in detail especially in the relevant case of Galois extensions.

The final part of the course shall focus on the units of ring of integers (proving the fundamental Dirichlet Theorem) and on class groups. Class groups are an extremely relevant invariant of number fields and their orders appear as special values of L-functions, which is one of the more active areas of research in modern number theory.

Practical Information

This is a 48 hours course (6 CFU), and takes place in the second semester. A basic reference for the course is:
D. Marcus *Number Fields*, Springer-Verlag 1977.

There will be an oral examination based on exercises and questions about the content of the course.

A few prerequisites are needed, namely a basic knowledge of Arithmetic, Commutative Algebra and Galois Theory.

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