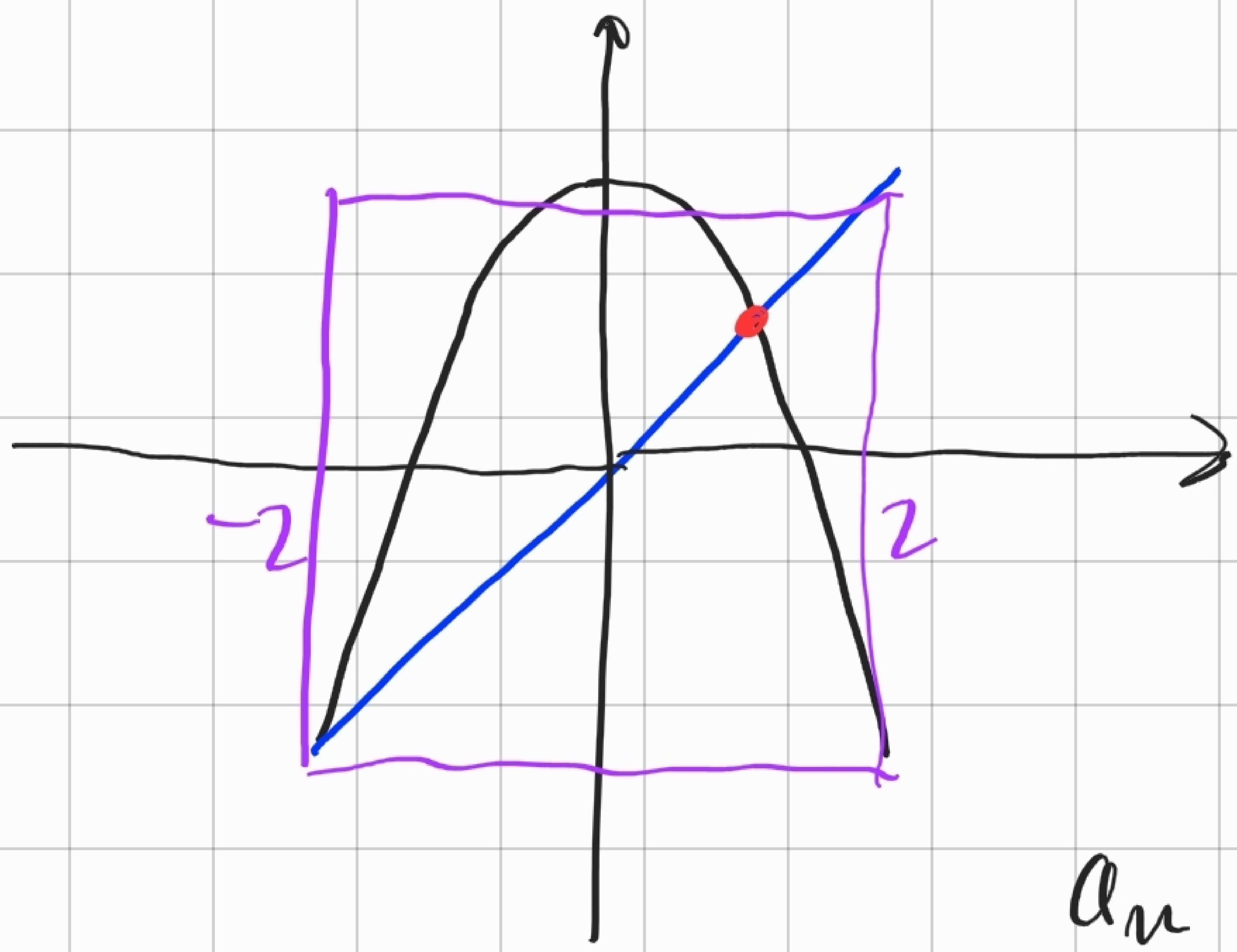
ANALISI MATEMATICA B
LEZIONE 33 - 9.12.2020

$$\begin{cases} a_0 = \alpha \\ a_{n+1} = 2 - a_n^2 \end{cases} \qquad f(x) = 2 - x^2$$



Mappa Logistica

$a_n \qquad b_n$

$[-2,2] \to [0,1]$

$$b_n = \frac{a_n + 2}{4} \qquad (a_n = 4b_n - 2)$$

$$b_{n+1} = \frac{a_{n+1} + 2}{4} = \frac{2 - a_n^2 + 2}{4}$$

$$= \frac{4 - a_n^2}{4} = \frac{4-(4b_n-2)^2}{4}$$

$$= \frac{\cancel{4} - 16b_n^2 - \cancel{4} + 16b_n}{4}$$

$$b_{n+1} = 4 \cdot b_n \cdot (1 - b_n)$$

<u>Crescita esponenziale:</u>

$$\begin{cases} a_0 = \alpha \\ a_{n+1} = c \cdot a_n \end{cases}$$

$a_0 = \alpha, \quad a_1 = c \cdot \alpha \quad a_2 = c^2 \cdot \alpha \ldots\ldots$

$\ldots\ldots \quad a_n = c^n \cdot \alpha.$

<u>Mappa logistica</u> 1 = popolazione.

$$a_{n+1} = c \cdot a_n \cdot (1 - a_n)$$

Abbiamo visto la volta scorsa $c = 4$.

$[0,1)$ è invariante?

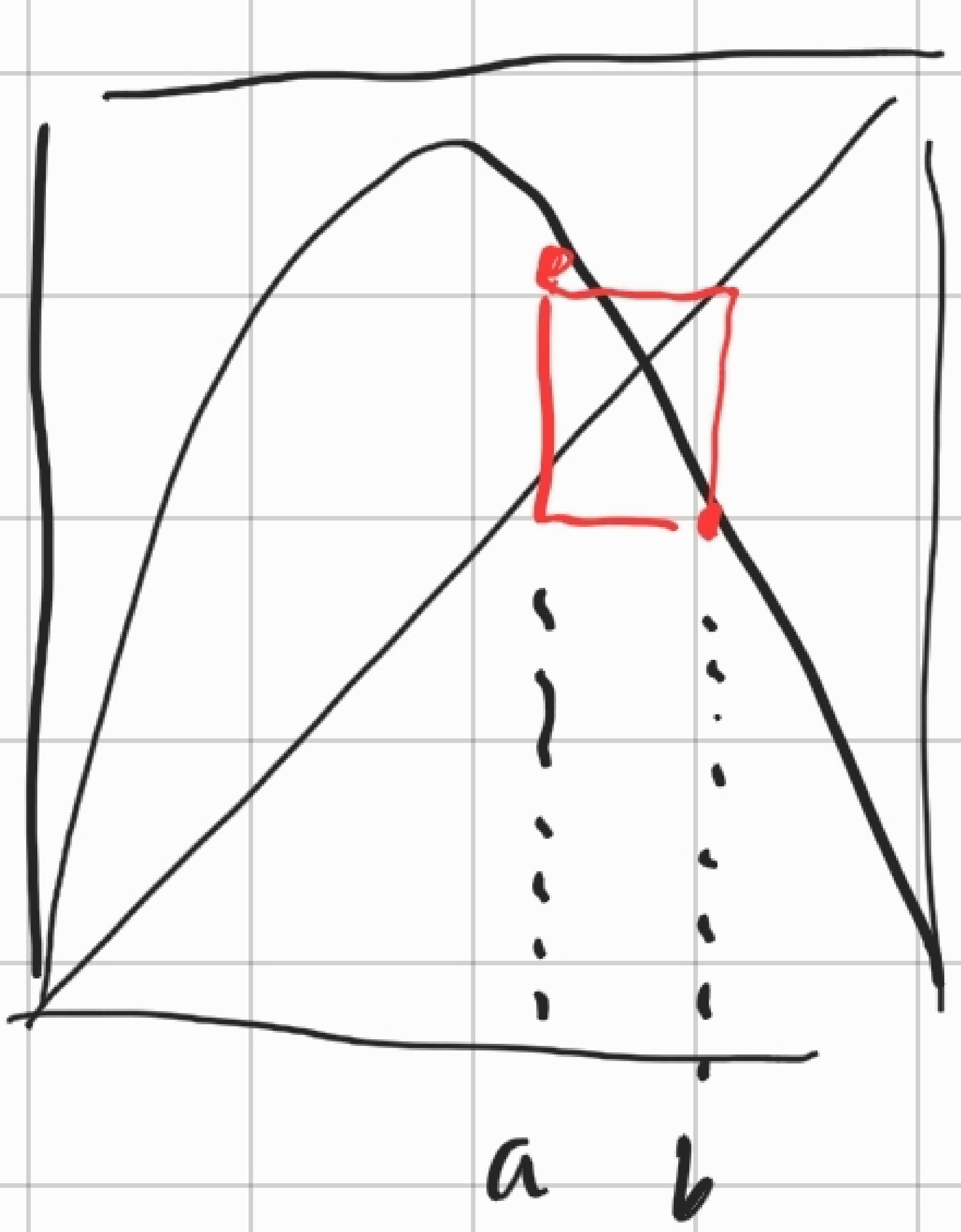
$$f(x) = c \cdot x(1-x)$$

se $x \in [0,1]$, $c \geq 0$ $f(x) \geq 0$

massimo di f in $[0,1]$ si ha per $x = \frac{1}{2}$.

$$f\left(\frac{1}{2}\right) = c \cdot \frac{1}{2}\frac{1}{2} = \frac{c}{4}.$$

$c \in [0,4]$ ok! l'intervallo $[0,1]$ è invariante.



$$\begin{cases} a_{2n} \to a \\ a_{2n+1} \to b \end{cases}$$

$$a_{2n+1} = f(a_{2n}) \rightarrow f(a) = b$$

$$a_{2n+2} = f(a_{2n+1}) \rightarrow f(b) = a$$

$$\begin{cases} f(a) = b \\ f(b) = a \end{cases}$$

$$f(f(a)) = f(b) = a$$

$$f(f(b)) = f(a) = b$$

a, b sono punti fissi di $\underline{\underline{f \circ f}}$.

$$f(x) = c \cdot x \cdot (1-x)$$

$$f(f(x)) = c\, f(x)\, (1 - f(x))$$

$$= c \cdot c \cdot x \cdot (1-x)\left(1 - c \cdot x\,(1-x)\right)$$

$$p(x) = c^2 x(1-x)(1-cx(1-x))$$

$p(x) = x$ è una eq. di grado IV.

$p(x) - x$ polinomio di grado $\overline{\underline{IV}}$

Siccome i punti fissi di f

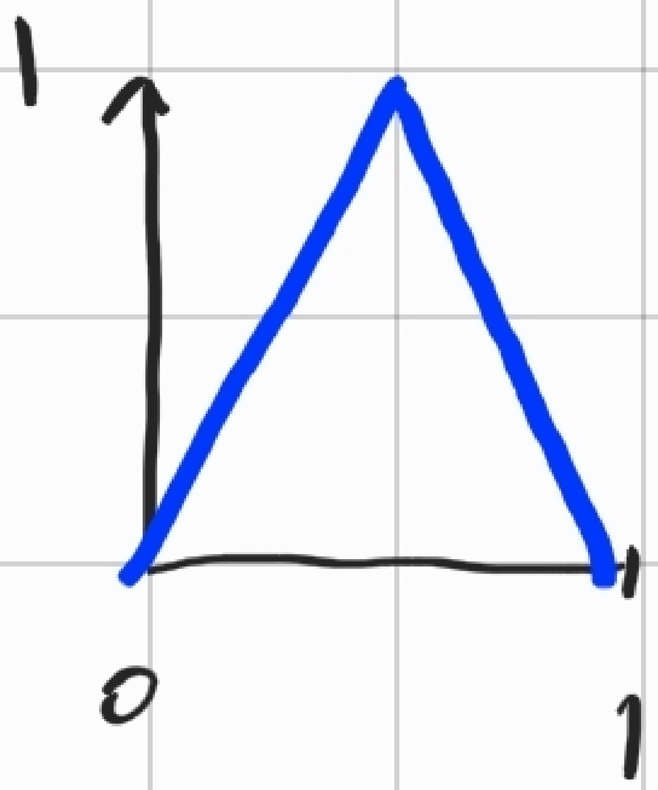
$$f(x) = x$$

sono anche punti fissi di $f \circ f$

gli zeri di $f(x) - x$

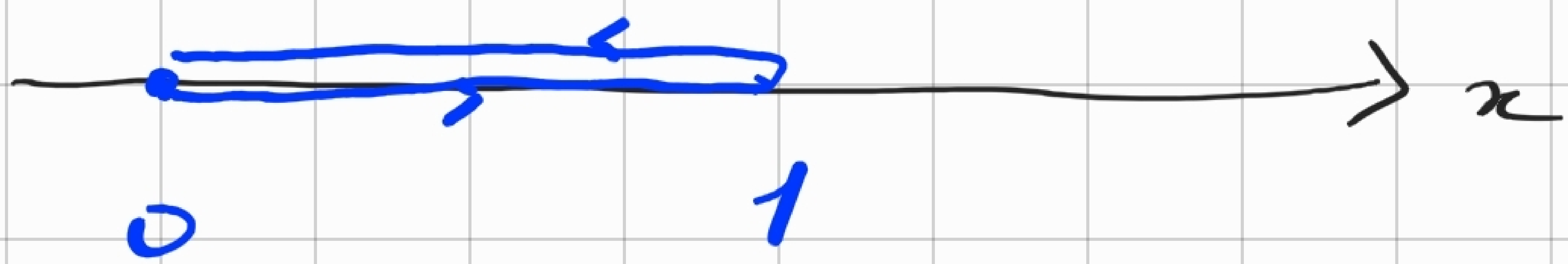
sono anche zeri di $p(x) - x$.

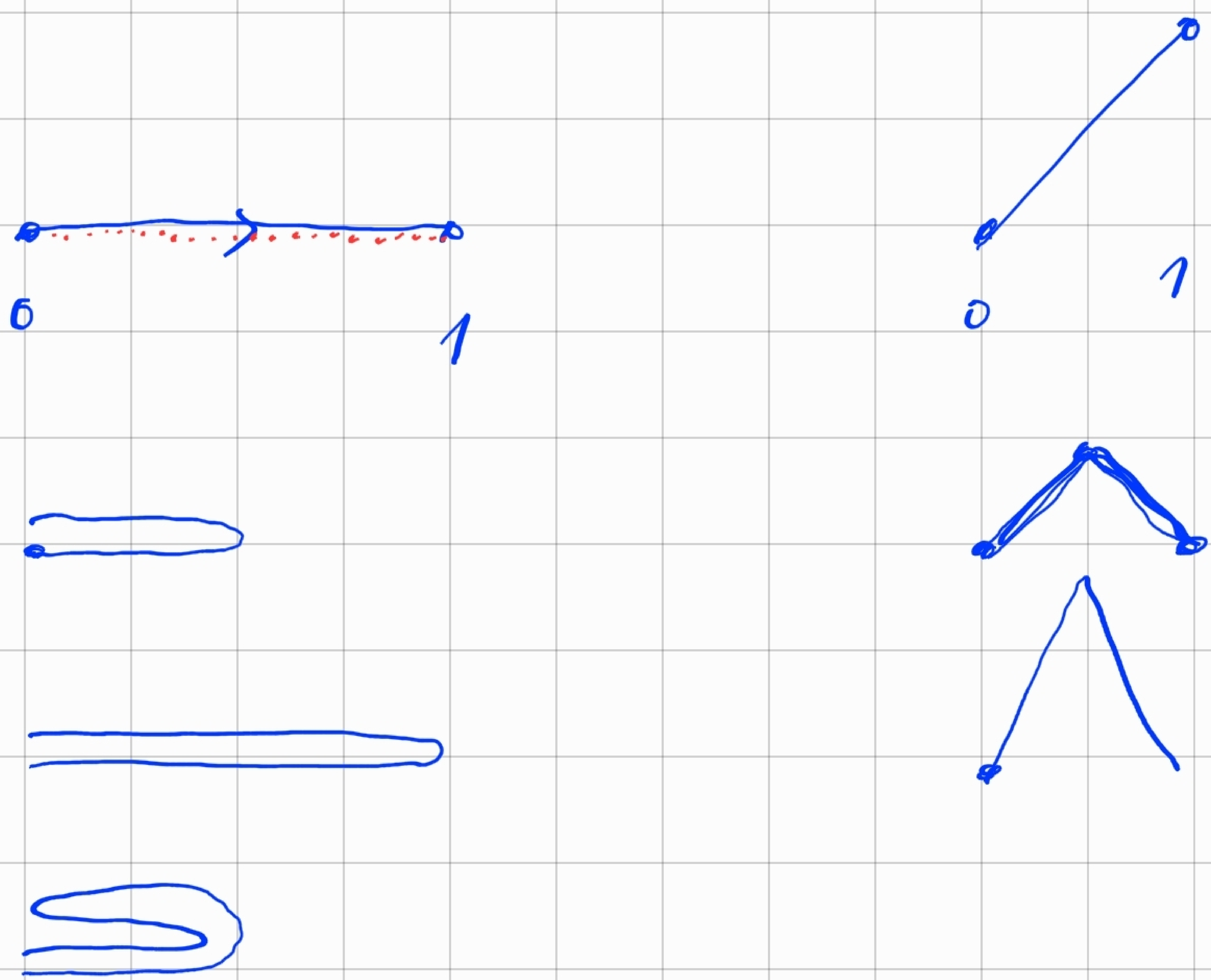
$p(x) - x$ è divisibile per $f(x) - x$.

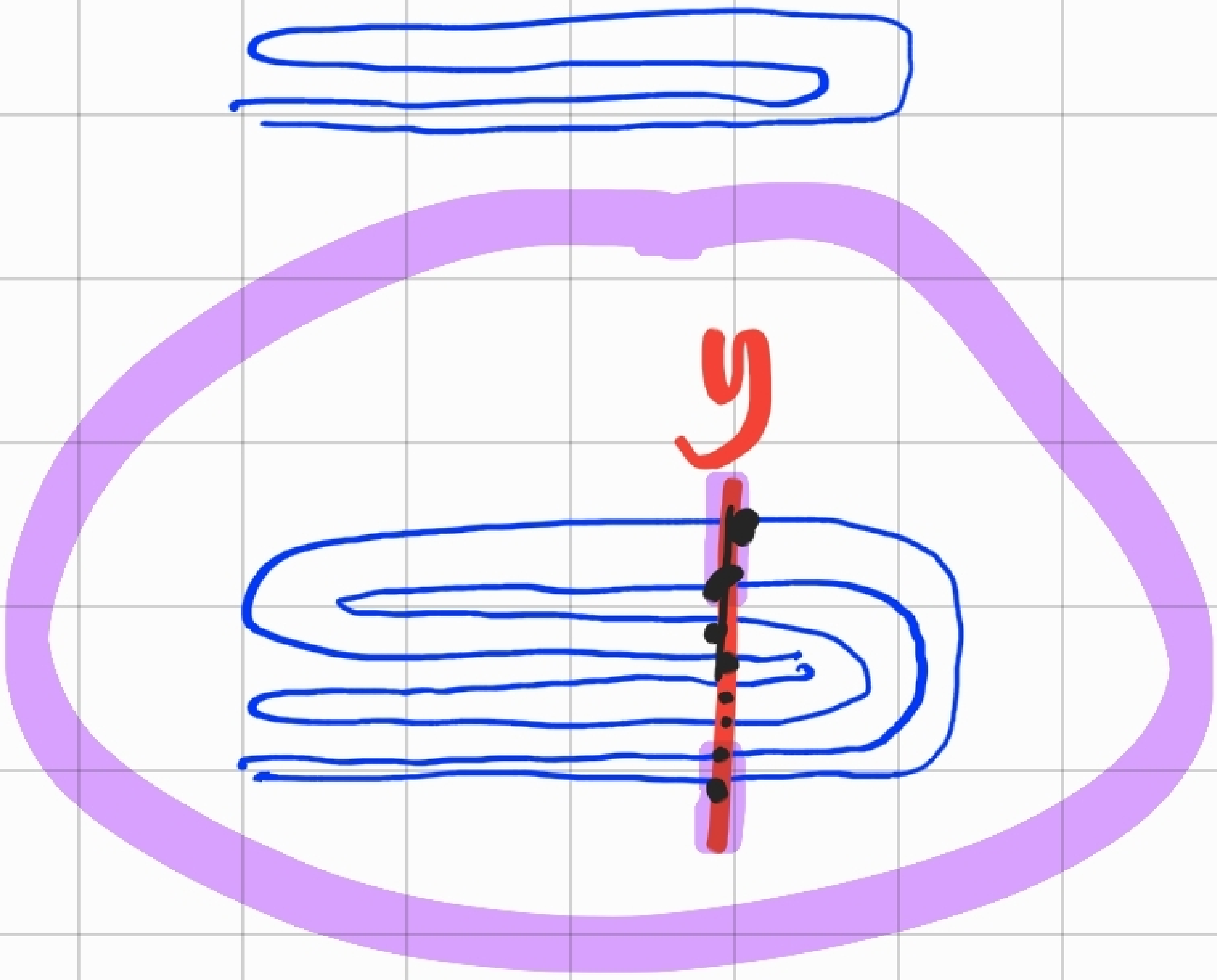


$$f(x) = \begin{cases} 2x & x \quad 0 \le x \le \frac{1}{2} \\ 2-2x & x \quad \frac{1}{2} < x \le 1 \end{cases}$$

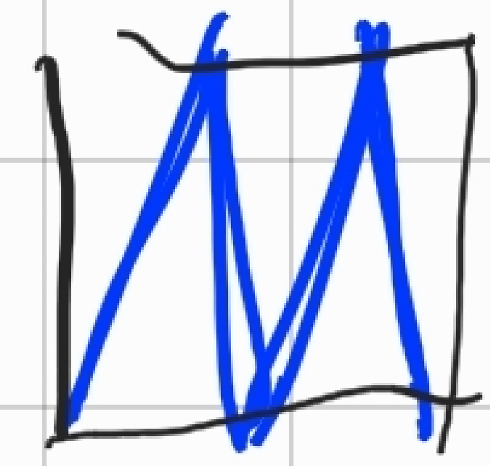
mappa a tenda
o mappa del panettiere



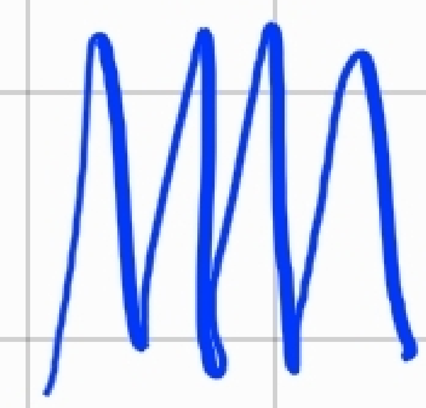




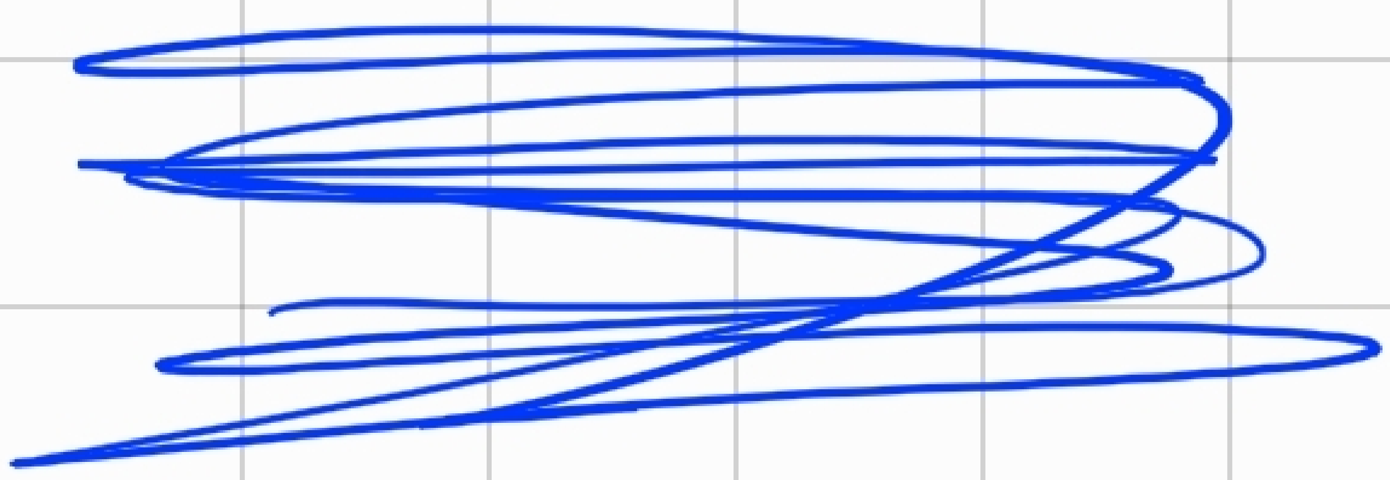
$f \circ f$



$f \circ f \circ f$



2^{n-1} strati



f^n

$$\begin{cases} a_0 = \alpha \\ a_{n+1} = f(a_n) \end{cases}$$

il sistema dinamico $a_n = f^n(\alpha)$

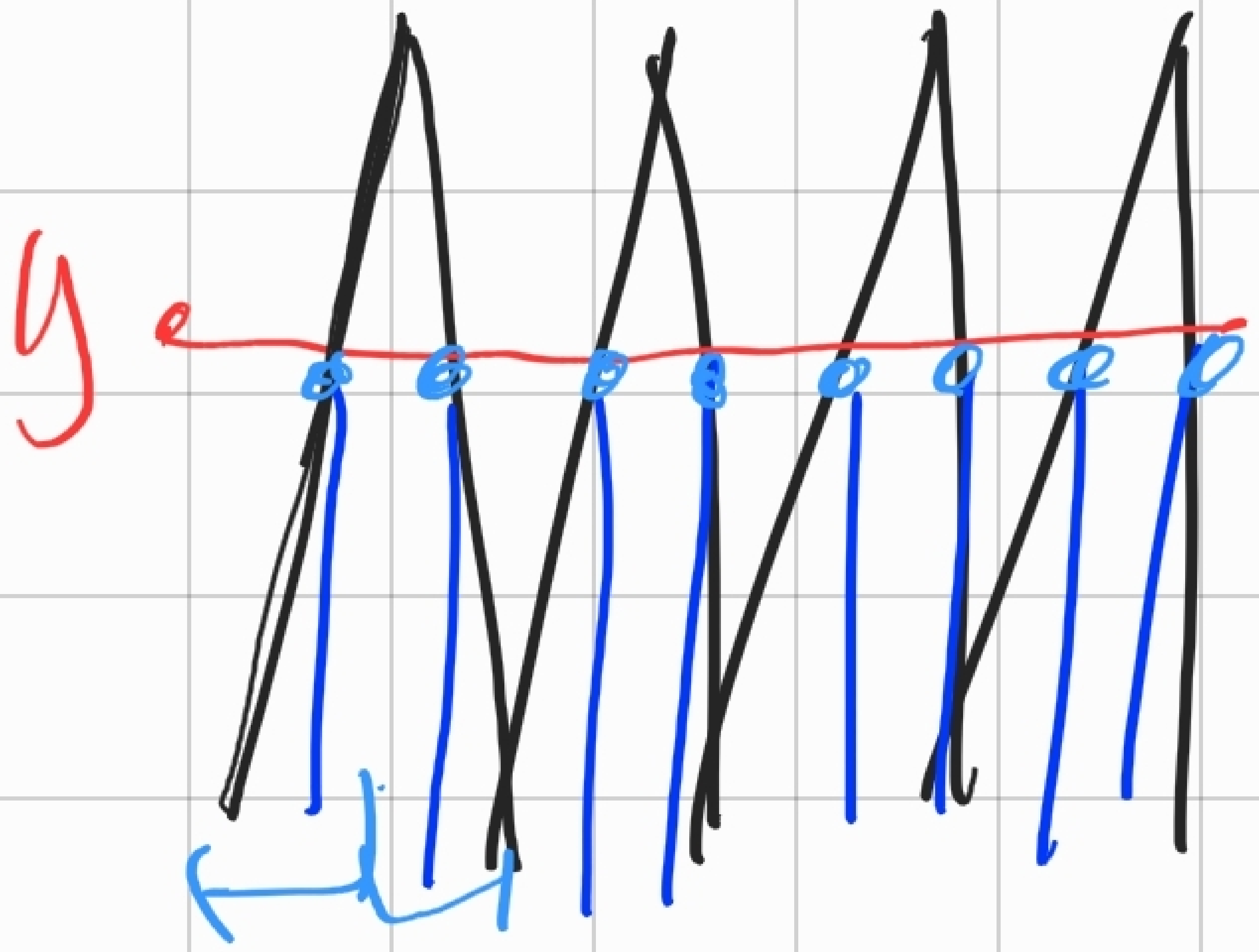
è "caotico" se è "topologicamente

transitivo":

Dati $x, y \in [0,1]$ dato $\varepsilon > 0$

esiste x' t.c. $|x - x'| < \varepsilon$ $\exists\, n$:

└ $f^n(x') = y$



$y = f^n(x)$

2^n punti

$\Leftrightarrow$

$\frac{1}{2^n} < \varepsilon$ se $n > \log_2 \frac{1}{\varepsilon}$

trovo x' vicino a qualunque x t.c.

$f(x') = y$ $\square$

Secondo metodo diagonale di Cantor

<u>Teorema</u> $\#\mathbb{R} > \#\mathbb{N}$

anzi $\#[a,b] = \#\mathbb{R} > \#\mathbb{N}$

(se $b > a$)

$\mathbb{N}\times\mathbb{N}$	0	1	2	3	4
0	0	2	5	9	14
1	1	4	8	13	
2	3	7	12		
3	6	11			
4	10				
	15				

$$\#(\mathbb{N}\times\mathbb{N}) = \#\mathbb{N}$$

<u>$\#\mathbb{Z} = \#\mathbb{N}$</u>

$$\#\mathbb{N} \le \#\mathbb{Q} \le \#(\mathbb{Z}\times\mathbb{N}) = \#\mathbb{N}$$

$$\#\mathbb{Q} = \#\mathbb{N}$$

dim $\#[0,1) > \#\mathbb{N}$

Sia $f : \mathbb{N} \to [0,1)$

$f(0) = 0,7531423218007 \ldots = 7 \cdot 10^{-1} + 5 \cdot 10^{-2} + 3 \cdot 10^{-3} \ldots$

$f(1) = 0,5732721$

$f(2) = 0,5555500$

$f(3) = 0,314159265 \ldots$

$\vdots$

$0,7751 \ldots$

modifico tutte le cifre.

$x = 0,2227 \ldots$

$\uparrow \uparrow \uparrow$

x non compare nell'elenco

$x = f(n)$ impongo l'n-esima cifra è diversa.

$$0,1999999\ldots = 0,2$$

$$\|$$

$$\frac{1}{10} + \sum_{k=2}^{+\infty} 9 \cdot 10^{-k}$$

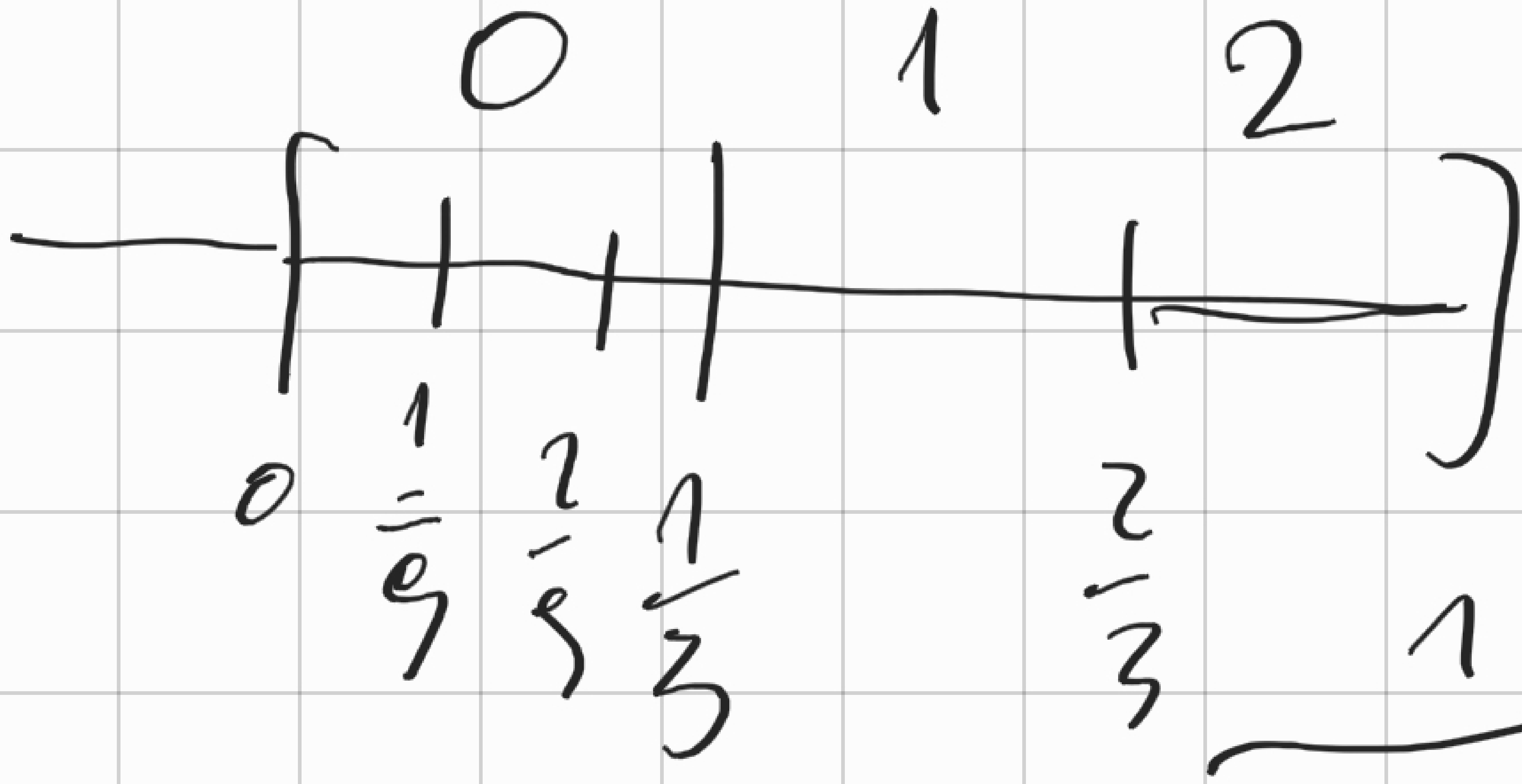
$$\Uparrow$$

$$= \frac{1}{10} + \frac{9}{100} \cdot \sum_{k=0}^{+\infty} \frac{1}{10^k}$$

$$= \frac{1}{10} + \frac{9}{100} \frac{1}{1 - \frac{1}{10}} \quad \ldots$$

Insieme di Cantor

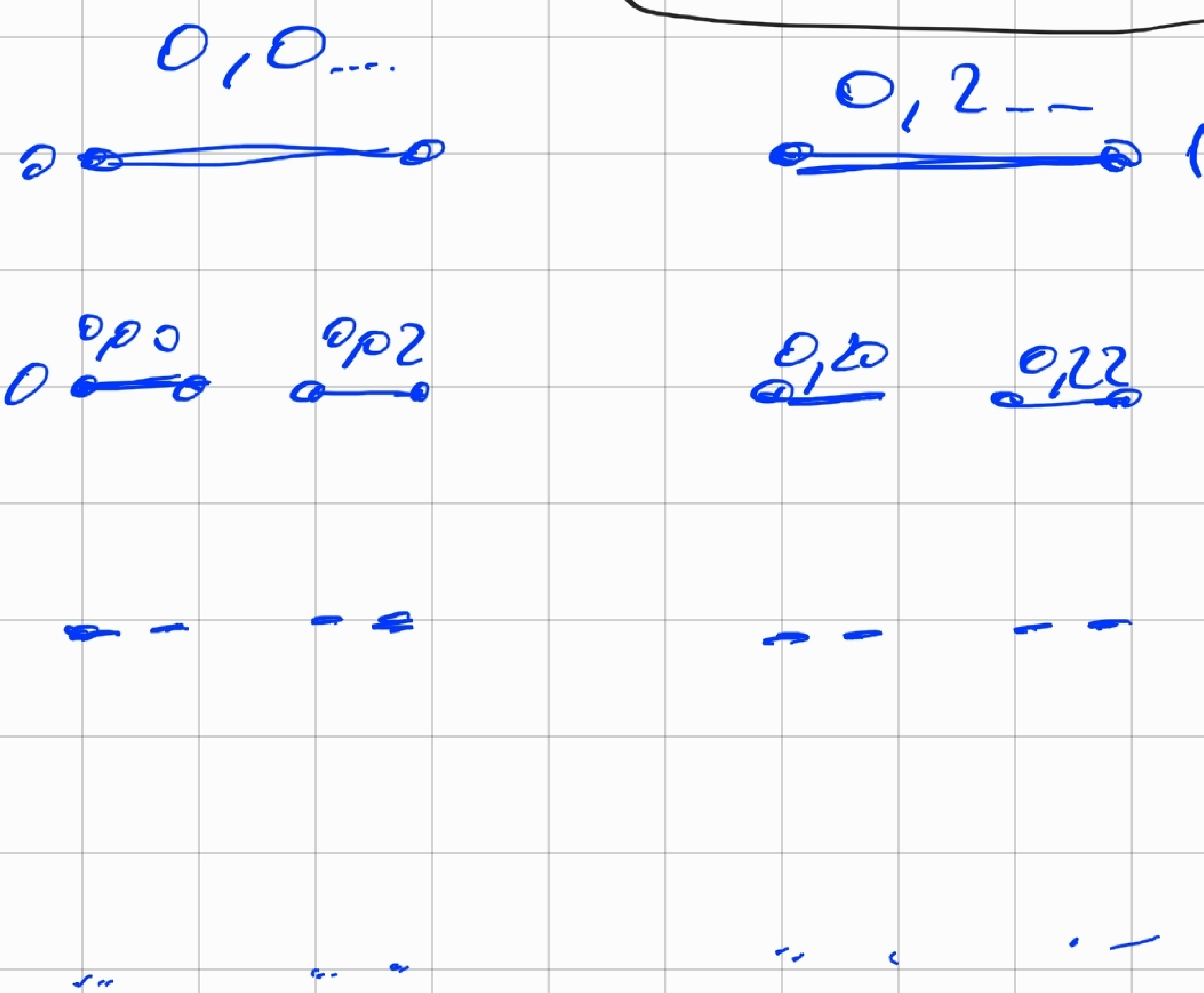
In base 3



0,1

0,2

C := i numeri che in base tre non hanno la cifra 1



$0{,}0022020022 2000$

$$\uparrow\uparrow\ \ \uparrow\ \ \ \ \uparrow\uparrow\uparrow$$

$\{$posizione delle cifre 2$\}$

$\subseteq \mathbb{N}$

$$\#C = \#\mathcal{P}(\mathbb{N})$$

$$\geq \qquad \qquad \parallel$$

$$\#\mathbb{R} \leq \# \mathcal{P}(\mathbb{Q})$$

$$\#\mathbb{R} = \#\mathcal{P}(\mathbb{N})$$

$$> \#\mathbb{N}$$
