Course Program
Heegaard Splittings and Handle Numbers

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The 30 hour course *Heegaard Splittings and Handle Numbers* will build from the foundations of the classical theory of Heegaard splittings of closed oriented 3-manifolds to the contemporary study of circle valued Morse functions and Morse-Novikov numbers of 3-manifolds. The course will touch upon several threads of low-dimensional topology and geometric topology. In particular, topics will involve relationships with Dehn surgery, analogies with bridge numbers and thin positions of knots, connections to modern Floer theories, interactions with sutured manifold theory, and the structures of curve complexes and Kakimizu complexes.

Due to the large number of Ph.D. students whose project concerns low-dimensional topology and geometric topology at the Math Department of the University of Pisa and at Scuola Normale Superiore. this course would be quite opportune for students in Pisa.

The plan of the course is laid out below, with additional topics as time permits. Each numbered item is allotted roughly 2.5 hours, adjusted as needed.

(1) Heegaard splittings of closed 3-manifolds, definitions and basic operations.
- Heegaard genus, basic examples
- Connected sums
- Heegaard splittings and stable equivalence from triangulations
- Heegaard splittings and stable equivalence from (real valued) Morse functions

(2) Heegaard diagrams
- Computations of basic invariants from Heegaard diagrams
- Operations on Heegaard diagrams: handle slides, stabilizations

(3) Heegaard splittings of 3-manifolds with boundary.
- Compression bodies
- Tunnel numbers of knots, manifolds
- Heegaard diagrams
- Heegaard splittings of Dehn fillings
- Tunnel numbers of connected sums

(4) Distance of Heegaard splittings
- The Curve Complex of a surface
- Weakly reducible and strongly irreducible Heegaard splittings

(5) Generalized Heegaard splittings
- Weak reductions and amalgamations
- Thin position of 3-manifolds

(6) Analogies in Knot Theory
- Bridge position and thin position
- Bridge numbers of connected sums
- Branched double covers

(7) Heegaard splittings of sutured manifolds
- Existence and stable equivalence
- Handle numbers of sutured manifolds
- Heegaard diagrams of sutured manifolds
- Sutured manifold decompositions, hierarchies, tautness
- Guts of sutured manifolds and handle numbers

(8) Circular (generalized) Heegaard splittings
- Circle valued Morse functions of 3-manifolds, knots
- Morse-Novikov number, handle number
- Morse-Novikov number of connected sums of knots

(9) Kakimizu complexes and Morse-Novikov numbers
- Definition of Kakimizu complexes and generalizations
- Handle enhanced Kakimizu complexes.
- Morse-Novikov genus of knots
- Goda's examples of phenomena and computations

(10) Analogies in Knot Theory
- Circular bridge position of links in solid tori, wrapping numbers
- Kakimizu complexes of links in solid tori
- 'Spinning' constructions and double branched covers
- Links in thickened tori and circular bridge positions

(11) Circular Heegaard splittings of sutured manifolds
- Thurston norm of 3-manifolds
- Handle number function of a sutured manifold
- Relation of (circular) handle number and tunnel number

(12) Nearly fibered knots
- Sutured Floer homology
- Guts and incompressible Seifert surfaces