Linear and Nonlinear hyperbolic systems

Many problems and model in physics or mechanics share the property of beeing hyperbolic. From a mathematical point of view, hyperbolicity relates stability properties of the time evolution to geometric properties of the equations. This course is intended to give an introduction for beginners to this field of hyperbolic equations. The first part will be devoted to the Cauchy problem, and the second to boundary value problems. The following items will be covered.

Part I : The Cauchy problem (6 lectures = 12 hours)

The first two lectures will be devoted to constant linear coefficients systems, using the Fourier synthesis which gives an immediate introductory insight to the main questions. The finite speed of propagation and the local theory of existence and uniqueness willl be covered. Examples from physics will be studied.

The second chapter (three lectures) will concern symmetric systems in the sense of Friedrichs. The main objective will be to introduce energy methods : a priori estimates, duality arguments, weak and strong solutions, finite speed, local theory. The third lecture will be devoted to nonlinear systems.

This part will end with an introductory survey of the extension to symmetrizable systems (one lecture).

Part II: Boundary value problems (6 lectures = 12 hours)

Again we will start with constant linear coefficients systems. The aim is to introduce the geometric aspects of the stability analysis, in particular the so-called Lopatinski condition (two lectures).

A lecture will be devoted to Friedrichs theory of dissipative boundary value problems.

In the final part of the course (three lectures) we will be concerned by Kreiss theory of symmetrizers. We will skip technical details to focus on the method. However, we will discuss the extension of Kreiss theory to cover some cases of systems with variable multiplicities.