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Report on the Research Activity

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This report summarises the activities carried out by the research fellow Irene Cavallari during the six months of activity from May 2020 to November 2020.

The research activity is supervised by Prof. Giovanni F. Gronchi and it is focused on the modeling of complex dynamical systems by patching different dynamics.

Research activity

The research activity carried out is about the study started during the six month between November 2019 and May 2020. The study concerns a dynamical system obtained by patching two different dynamics to model a more complex problem, that is the short-period evolution of near Earth satellites subjected to the solar radiation pressure. Since the satellite is screened from the Sun while it crosses the Earth shadow region, here we can adopt Kepler's dynamics. By neglecting the apparent motion of the Sun and by assuming the solar radiation pressure force constant, we adopt Stark's dynamics in the out-of-shadow region. We consider the planar case in which the solar radiation pressure lies on the plane of motion of the satellites and we model the shadow as a half-stripe.

The most recent outcomes of the research are about a Poincaré map, called Sun-shadow map, defined to study the dynamics of the problem. The Poincaré section used corresponds to the upper boundary of the shadow region in the configuration space; we consider trajectories leaving the section in the outward direction with respect to the shadow region. Traditionally, the Poincaré maps of autonomous two-dimensional Hamiltonian systems are defined by fixing the value of the Hamiltonian. This is not possible for this problem, since the Hamiltonian is not conserved along the flow. However, here we can fix the value of projection of the generalized Laplace-Lenz vector onto the direction of the solar radiation pressure acceleration, $\mathscr{L}_s$. Indeed, even though $\mathscr{L}_s$ is not a constant of motion, it assumes the same value in Stark's regime before and after the satellite crosses the shadow. The following progresses were made:

- we showed that the map is not area-preserving;
- we numerically determined the domain and the image of the map;
- we determined a numerical technique to compute the Jacobian matrix of the map based on [Simo, 1989];
- we observed several islands and periodic points of the map;
- we numerically computed the invariant manifolds of the Sun-shadow map fixed point, existing for a specific interval of $\mathscr{L}_s$ values.

To compute the fixed point invariant manifolds, we developed a technique based on both [Hobson, 1993] and [Lega and Guzzo, 2016]. The Melnikov method to determine eventual homoclinic points between the stable and the unstable manifold was studied. However, it seems not to be applicable to the Sun-shadow map.

Currently, we are formalizing the results obtained in a paper to be submitted for publication in a scientific journal.

During the same period (May-November 2020), I started collaborating in a group project with other researches involved in the Stardust-R project. The project goal is to design a constellation of satellites around the Moon to be employed for a positioning system, similar to the Global Positioning System (GPS) used for the navigation on Earth. We have selected an optimal Walker constellation assuring a global coverage of the Moon, with good values of the Geometric Dilution of Precision (a parameter used to evaluate the impact of the constellation geometry on the position error). Since the south polar region is the currently selected target of the future crewed mission on the Moon, the NASA Artemis mission, we also determined an optimal constellation for a local positioning system covering the latitudes between $-90°$ and $-55°$. The constellation is inspired to a Walker one, but with elliptical, frozen orbits. The plan is to analyze the stability of these constellations to evaluate the station keeping costs and to determine whether a GPS system around the Moon is feasible.

1 Other Activities

I helped in the organization of the Global Virtual Workshop I, held virtually on 7-10 September 2020. During the event I presented both my research on the Sun-shadow problem and the first outcomes of the group project.

I passed the exams of the following courses:

- Elementi di calcolo della variazioni by Professor A. Pratelli, on June 15th;
- Determinazione Orbitale, by Professor G. F. Gronchi, on July 2nd.

from September, I am attending the following courses:

- Complementi di Analisi Funzionale, by Professor A. Pratelli;
- Teoria e Metodi dell'Ottimizzazione, by Professor G. Bigi.

References

[Hobson, 1993] Hobson, D. (1993). An Efficient Method for Computing Invariant Manifolds of Planar Maps. *Journal of Computational Physics*, 104:14–22.

[Lega and Guzzo, 2016] Lega, E. and Guzzo, M. (2016). Three-dimensional representations of the tube manifolds of the planar restricted three-body problem. *Physica D*, 325:41–52.

[Simo, 1989] Simo, C. (1989). On the Analytical and Numerical Approximation of Invariant Manifolds. In Frontières, E., editor, *Modern Method in Celestial Mechanics*, pages 285–329. Benest, D., Froeschle, C., Observatoire de la Côte d'azur.