

# Report Research Activity

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This report summarises the activities carried out by the research fellow during the first six months of activity, from May 25th 2021 to November 24th 2021.

The research activity is supervised by Prof. Giovanni F. Gronchi and Phd Giulio Baù and it is related to techniques for Initial Orbit Determination (IOD) of space objects from observations. In particular it is focused computation of several asteroid orbits with the observations of the Isolated Tracklet File (ITF) available at the Minor Planet Center and the application of the IOD to asteroids and space debris.

This activity belongs to the European project Stardust-R and it is founded by an MSCA-ITN-ETN - European Training Networks (see <http://www.stardust-network.eu/> for more information).

## Developments and work in progress:

We have mainly followed the work presented in the research papers of Orbit Determination with the two-body integrals I, II, III (see [1, 2, 3]) and the book by A. Milani and G. F. Gronchi [6]. In particular, our work was focused on the use of the polynomial method (see [3]) with the aim of identify whether if two tracklets belong to the same observation and to construct a preliminary orbit (link2 problem) and similarly on the use of the method introduced in [4] with the aim to identify whether if three tracklets belong to the same observation and to construct a preliminary orbit (link3 problem).

To this purpose we have made use of the software Orbfitter (<http://adams.dm.unipi.it/orbfit/>), introducing some improvements in the implementation of said methods. In addition, thanks to the datasets provided by Robert Jedicke, these methods have been tested and analysed in various controlled conditions. We have made some improvements in the implementation of these methods and we are trying to use different techniques in order to recover lost solutions. For these datasets, we have found quite good results in terms of the total number of real identifications obtained.

An exploration of the ITF was done using a strategy combining the link2 and link3 methods mentioned previously.

During October 2021 Professor Robert Jedicke visited us for 3 weeks. On this visit we began to write a paper that we will be finished soon. This work aims to study the numerical behaviour of the Keplerian Integral methods for the initial orbit determination problem. We also plan to write another paper regarding the application of these

methods to the ITF.

In addition we analyse the work [7] of Mossotti for the orbit determination problem and we construct a topocentric version of the method that leads to better results. The performance of the two versions are compared through numerical tests with synthetic asteroid data using different time intervals between consecutive observations and different astrometric errors. We also compare the topocentric method with Gauss's method using simulated observations with the expected cadence of the VRO-LSST telescope. The results of these methods can be found in [5].

## Activities:

I have attended to the following schools and workshops:

- Barcelona UB-UPC Dynamical Systems Group Seminar.
- XXVI Congress of Differential Equations and Applications /XVI Congress of Applied Mathematics (CEDYA/CMA), June 14-18, 2021.
- International Conference on Nonlinear Mathematics and Physics, June 30 - July 2, 2021.
- Summer School on Qualitative Theory of Piecewise Ordinary Differential Equations, July 12-16, 2021.
- Jornadas de Trabajo en Mecánica Celeste, August 30 - September 1, 2021.
- Stardust-R Global Virtual Workshop II, September 13-17, 2021.
- IAU364 - Multi-scale (time and mass) Dynamics of Space Objects, October 18-22, 2021.

## References

- [1] Gronchi, G.F., Dimare, L., Milani, A.: *Orbit determination with the two-body integrals*. Cel. Mech. Dyn. Astron. 107(3), 299–318 (2010)
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- [3] Gronchi, G.F., Baù G., Marò S.: *Orbit determination with the two-body integrals. III*. Cel. Mech. Dyn. Astron. 123, 105–122 (2015)
- [4] Gronchi, G.F., Baù G., Milani A.: *Keplerian integrals, elimination theory and identification of very short arcs in a large database of optical observations*. Cel. Mech. Dyn. Astron. 127, 211–232 (2017)

- [5] Gronchi, G.F., Baù, G., Rodríguez, O., Jedicke, R., Moeyens, J. : *Generalization of a method by Mossotti for initial orbit determination*. Cel. Mech. Dyn. Astron, 133. Available at <https://www.doi.org/10.1007/s10569-021-10038-4> (2021)
- [6] Milani, A., Gronchi, G.F.: *The Theory of Orbit Determination*. Cambridge University Press, Cambridge (2010)
- [7] Mossotti O.F., *Nuova analisi del problema di determinare le orbite dei corpi celesti*, Pisa, Domus Galileana (1816-1818)

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