Semi-annual report 01/04 – 30/09/2020

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The work done so far in the framework of the research grant "***Mathematical Logic: models, sets, computability, applications in combinatorics, transseries and exponential fields***" focuses on the structure theory of sets and groups definable in various important model-theoretic settings. In the newly submitted article (4) below, we extend the influential Pillay's Conjecture for definable groups from the o-minimal setting to the weakly o-minimal non-valuational setting. In the revised article (5), we develop dimension theory in the setting of expansions of o-minimal structures expanded by a derivation. In the current work (6), we combine Eleftheriou's previous expertise [4], together with that of Berarducci-Mamino [1], to obtain a new, orthogonal decomposition of definable groups in arbitrary o-minimal structures.

Below we report on papers that were accepted, submitted/revised, or progressed during 01/04–30/09/2020:

Papers accepted:

(1) P. Eleftheriou, Counting algebraic points in expansions of o-minimal structures by a dense set, *Quarterly Journal of Mathematics* (to appear), arXiv:1708.03936v3, 15 pages.

(2) P. Eleftheriou, Small sets in Mann pairs, *Archive for Mathematical Logic* (to appear) arXiv:1812.07970, 9 pages.

(3) P. Eleftheriou, A. Günaydin, P. Hieronymi, The Choice Property in tame expansions of o-minimal structures, *Mathematical Logic Quarterly* 66 (2020), 239–246.

Papers submitted/revised:

(4) P. Eleftheriou, Pillay's conjecture for groups definable in weakly o-minimal non-valuational structures, arXiv: 2001.08209. (Submitted to *Bulletin of the London Mathematical Society* in May 2020).

Let G be a group definable in a weakly o-minimal non-valuational structure $\mathcal{M}$. Then G/G^{00}, equipped with the logic topology, is a compact Lie group, and if G has finitely satisfiable generics, then $\dim(G/G^{00}) = \dim(G)$. Our main technical result is that G is a dense subgroup of a group definable in the canonical o-minimal extension of $\mathcal{M}$.

(5) P. Eleftheriou, O. Leon Sanchez, N. Regnault, On coincidence of dimensions in closed ordered differential fields, arXiv:2002.12929. (Submitted to the *Notre Dame Journal of Formal Logic* in February 2020, revised version September 2020)

Let $\mathcal{K} = \langle \mathcal{R}, \delta \rangle$ be a closed ordered differential field, in the sense of Singer [5], and C its field of constants. In this note, we prove that, for sets definable in the pair $\mathcal{M} = \langle \mathcal{R}, C \rangle$, the δ-dimension from Brihaye-Michaux-Rivière [2] and the large dimension from Eleftheriou-Günaydin-Hieronymi [3] coincide. As an application, we characterize the definable sets in $\mathcal{K}$ that are internal to C as those sets that are definable in $\mathcal{M}$ and have δ-dimension 0. We further show that, for sets definable in $\mathcal{K}$, having δ-dimension 0 does not generally imply co-analyzability in C (in contrast to the case of transseries). We also point out that the coincidence of dimensions also holds in the context of differentially closed fields and in the context of transseries.

Papers in progress:

(6) A. Berarducci, P. Eleftheriou, M. Mamino, Orthogonal decomposition of o-minimal groups (tentative title).

We explore groups definable in the disjoint union of finitely many structures. We prove that under certain conditions, every such group 'splits'; namely, it is a product of sets definable in the respective structures. As an application, we obtain a decomposition theorem for groups definable in arbitrary o-minimal structures: every such group is a product of orthogonal subsets, which can no further be decomposed into orthogonal subsets.

References

[1] A. Berarducci, M. Mamino *Groups definable in two orthog- onal sorts*, Israel Journal of Mathematics, 208 (2015), 413–441.

[2] T. Brihaye, C. Michaux and C. Rivière, *Cell decomposition and dimension function in the theory of closed ordered differential fields*, Annals of Pure and Applied Logic 159 (2009), 111–128.

[3] P. Eleftheriou, A. Günaydin and P. Hieronymi, *Structure theorems in tame expansions of o-minimal structures by dense sets*, Israel Journal of Mathematics (to appear), arXiv:1510.03210v4, 46 pages.

[4] P. Eleftheriou, Y. Peterzil, J. Ramakrishnan, *Intrepretable groups are definable*, Journal of Mathematical Logic, Vol. 14, No. 1 (2014), 47 pages.

[5] M. Singer, *The model theory of ordered differential fields*, Journal of Symbolic Logic 43 (1978), 82–91.