Report on the Research Activity

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This report summarises the activities carried out by the research fellow during the first six months of activity, from November 2019 to May 2020.

The research activity is supervised by Prof. Giovanni Gronchi and it is focused on the modeling of complex dynamical systems by patching different dynamics.

Research activity

The research activity concerns the study of dynamical systems made by different patched dynamics, which may be exploited to model more complex dynamical system in first approximation.

The currently analyzed problem concerns the evolution of near Earth celestial objects trajectories subjected to the solar radiation pressure, taking into account the effects of the Earth shadow. In the shadow, the solar radiation pressure force doesn't act and it is possible to assume a Keplerian model for the dynamics. The problem has been simplified by considering a planar motion. Moreover, as suggested by Beletsky [1], the apparent solar motion has been neglected. The revolution period of the Sun is highly superior to the orbital period of near Earth celestial objects. Thus, over short time intervals, during which these objects may perform several revolutions depending on their distance from the central body, the Sun position shift is negligible. By considering the Sun position fixed, the solar radiation pressure force can be assumed as constant: a Stark dynamics can be assumed [2]. The work performed consists in the study of the dynamical system made by patching a Stark dynamics outside the shadow region with a Keplerian dynamics inside the shadow area. The shadow has been modeled as a half-stripe.

The following progresses have been done:

- a transformation of variables has been adopted, allowing to obtain a regularization similar to the Levi-Civita one and the variables separation;
- the existence of a family of periodic orbits of brake type has been demonstrated;
- the Sun-Shadow map has been built: it is a Poincaré map used for the study of the dynamics;
- the existence of a first integral for the map has been demonstrated: it corresponds to the projection of the generalized Laplace Lenz integral [3], L_s, onto the direction of the constant solar radiation pressure force;
- it has been demonstrated that the Sun-Shadow map is differentiable.

The Sun-Shadow map has been built by exploiting a Poincaré section in the phase space corresponding to upper boundary of the shadow area with the motion directed to exit from the shadow. The value of L_s integral is fixed. The map shows the existence of regular and chaotic regions. In particular, for a specific interval of L_s values, the existence of fixed elliptic points has been observed and the approximated position of the fixed hyperbolic point corresponding to the periodic orbit of brake type has been identified. Currently, techniques to exploit the Fast Lyapunov Indicators (FLI) to reconstruct the manifolds of periodic orbits are studied: they can be adopted in order to visualizes the separatrix of the hyperbolic fixed point in the map[4].

An other dynamical system consisting of two patched dynamics has been analyzed. It is a simple toy model which has been useful to understand how address the study of a more complex dynamical system made by patched dynamics, as the one introduced. It consists in the patching of two different Keplerian dynamics: in a Cartesian plane, the positions of two central bodies are symmetric with respect to the y axis where the patching is performed. A Poincaré map has been built by considering as Poincaré section, the surface $\Sigma = \{(x, y, \dot{x}, \dot{y}) : x = 0, \dot{x} > 0\}$. Even if the energy is not constant in time, it is a first integral for the map if the two central bodies have the same mass. Thus, an energy value one has been fixed for the construction of the map. If the energy is negative, the map shows regular motion; two fixed symmetric elliptic points have been identified. The map curves regularity suggested the existence of another integral of motion, that has been determined. If the two central bodies have different masses, the energy is not a constant for the map. Thus, the map has been built by fixing the initial value of the energy. The fixed elliptic points disappear and the map curves overlap. However, their regularity suggested the existence of a first integral that has been determined.

1 Other Activities

I attended the following schools and workshops:

- Stardust-R Opening Training School, University of Strathclyde, Glasgow, 2-7 December 2019;
- I-CELMECH Training School, Università degli Studi di Milano, Milano, 3-7 February 2020;
- Stardust-R Training School II, Politecnico di Milano, Milano, 10-13 February 2020;
- Stardust-R Local Training Workshop I, organised by the Technical University of Madrid and Deimos, 18-21 May 2020.

I attended the following courses:

- Istituzioni di Fisica Matematica, by Professor G. F. Gronchi;
- Determinazione Orbitale, by Professor G. F. Gronchi;
- Elementi di calcolo della variazioni by Professor A. Pratelli.

References

[1] Beletsky V.V. *Essays on the motion of celestial bodies.* Birkhäuser, 2001.

[2] Beletsky V.V. Space- flight trajectories with a constant-reaction acceleration vector. *Cosmic Research,* 2(3):345–350, 1964.

[3] Redmond P.J. Generalization of the runge-lenz vector in the presence of an electric field. *Phys. Rev.*, 133(5B):1352–1353, 1964.

[4] Lega E. Guzzo M. Three-dimensional representations of the tube manifolds of the planar restricted three-body problem. *Physica D: Nonlinear Phenomena,* 352:41–52, 2016.

G F Gronchi