INFINITE ERGODIC THEORY WITH APPLICATIONS TO ELEMENTARY NUMBER THEORY

This course will follow, albeit in reduced form, the book "Infinite Ergodic Theory of Numbers" by Kesseböhmer, Munday and Stratmann.

1. Number-theoretic dynamical systems

Here we will introduce our two main examples, the Gauss map and the Farey map, give a brief review of continued fractions, and mention some other examples that we will return to in later lectures.

2. Review of basic ergodic theory

Here we review basic concepts such as invariant measures, recurrence and conservativity, transfer operators, ergodicity, and exactness. We will compare the situation for the more “classical” probability-measure-preserving systems with the infinite measure setting. We will also introduce the technique of inducing for infinite-measure-preserving systems, and show how it can be used to give an elegant proof, originally due to Zwiemüller, of Hopf’s ergodic theorem.

3. Infinite ergodic theory

Here we will formulate and prove the very general Chacon-Ornstein ergodic theorem, and investigate pointwise dual ergodicity and ψ-mixing.

4. Connecting classical probability theory and infinite ergodic theory: sum-level sets

Using the number-theoretic examples introduced at the beginning of the course, we will illustrate how applying a reasonable dose of infinite ergodic theory yields interesting results in the theory of continued fractions. We will also clarify these useful notions have their roots in the classical renewal theory developed (mostly) in the 1960s.