

Corso di Dottorato in Matematica
Mini Corso di Dottorato a.a. 2021-2022

Docente: **Giovanni Paolini**

Titolo: **Combinatorial topology and group theory**

Abstract:

In this course, we will cover combinatorial techniques to study topological spaces and groups. On the topological side, we are going to see how discrete Morse theory can be used to prove homotopical and homological statements through the language of partially ordered sets (posets). On the group-theoretic side, we are going to introduce Garside theory and apply it to the study of symmetric groups and braid groups (as well as their generalizations, Coxeter and Artin groups). Using discrete Morse theory, we will prove one of the main results of Garside theory, namely the construction of a classifying space (or $K(\pi,1)$) for an arbitrary Garside group. We will also mention algorithmic aspects in discrete Morse theory (related to collapsibility) and in Garside theory (the word problem). If time permits, we will also touch on current research directions in the theory of Garside groups, Coxeter groups, and Artin groups.

Prerequisiti:

Basic algebraic topology and homological algebra (e.g., the UniPi courses "Elementi di topologia algebrica" and "Istituzioni di algebra"). Emanuele Delucchi's course on "Combinatorial methods in topology" (Spring 2021) is not a requirement but will be useful.

Programma provvisorio in 5 lezioni:

1. Partially ordered sets and relevant examples
2. Topological discrete Morse theory, applications and algorithms
3. Algebraic discrete Morse theory
4. Combinatorial Garside structures, the word problem, and examples (symmetric groups and braid groups)
5. Classifying spaces of Garside groups and consequences