

PhD Course: Yangians in geometry and representation theory

Instructors.

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Dates.

March – May, 2021 (around 30 hours)

Description.

The course will be divided in three parts. The first part will describe the algebraic theory of Yangians from their definitions (motivated by mathematical physics), their several presentations, and their category of finite-dimensional representations. The second part will focus on Maulik-Okounkov's approach to Yangians based on the theory of stable envelopes. Finally, in the last part we will study in detail Maulik-Okounkov's construction in the case of the cotangent of flag varieties.

Syllabus.

Part one (Appel): Drinfeld's approach to Yangians

- Motivations from Mathematical Physics: lattice models
- The Yangians of $\mathfrak{gl}(N)$ and $\mathfrak{sl}(N)$
- The Yangian of a Kac-Moody algebra and Drinfeld's "new presentation"
- Finite-dimensional representations
- The meromorphic R-matrix
- An explicit example: Varagnolo's action of Yangians on cohomology of flag varieties

Part two (Sala): Stable Envelopes and Maulik-Okounkov Yangians

- Torus actions, Bialynicki-Birula decompositions
- Equivariant cohomology, localization theorem

- Stable envelopes in equivariant cohomology
- Maulik-Okounkov R-matrix

Part three (Lanini): Stable envelopes and cohomology of cotangent bundles of flag varieties

- Maulik-Okounkov's theory in the case of the cohomology of cotangent bundles of flag varieties

Essential Bibliography.

- Carrell, Torus Actions and Cohomology
- Maulik and Okounkov, Quantum groups and quantum cohomology (<https://arxiv.org/pdf/1211.1287.pdf>)
- Molev, Yangians and classical Lie algebras
- Chari and Pressley, A guide to quantum groups
- Gautam, Toledano Laredo, and Wendlandt, The meromorphic R-matrix of the Yangian (<https://arxiv.org/abs/1907.03525>)
- Su, Stable Basis and Quantum Cohomology of Cotangent Bundles of Flag Varieties (<https://www.math.toronto.edu/csu/thesis.pdf>)