Ph.D. course

Stochastic analysis on Gaussian and Poisson spaces, functional and geometric applications

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Abstract

Part 1: Gaussian analysis. The first part of this course is an introduction to the theory of Gaussian approximations in Wiener chaos and its applications. The core of this Gaussian analysis is the fruitful combination of two probabilistic techniques, namely Malliavin calculus and Stein's method, that appeared first in the article *Stein's method on Wiener chaos* by Nourdin and Peccati (Probab. Theory Related Fields 2009) and then enormously developed. We will present some basic elements of both Stein's method and Malliavin calculus and see how to combine them in order to provide a complete characterization of Central Limit Theorems on a fixed Wiener chaos, in terms of fourth-moment-conditions. The latter control also the rate of convergence in various probability metrics such as Wasserstein and Total Variation distances. If time permits we will discuss some applications, e.g. CLTs for zeros of random polynomials. (**M. Rossi**)

Part 2: Poisson analysis. The aim of the second part of this course is to present some basic elements of stochastic analysis for non-linear functionals of a random Poisson measure. We will adopt a point of view combining the Gamma calculus for Markov generators, Malliavin calculus techniques, and Mecke-type formulae on configuration spaces. After having described some basic functional inequalities related to the concentration of measure phenomenon (in particular, modified logarithmic Sobolev inequalities), we will show how these techniques can be used in order to study the fluctuations of random geometric objects, with particular emphasis on quantities emerging in the theory of random geometric graphs. The main highlight of the last part of the course will be a study of second-order Poincaré inequalities on the Poisson space, yielding a fully quantitative version of the theory of geometric stabilization initiated by Kesten, Lee, Penrose and Yukich at the end of the 90s.(**G. Peccati**)