

# Report on the Research Activity

Irene Cavallari

30/04/2021

This report summarises the activities carried out by the research fellow Irene Cavallari during the six months of activity from November 2020 to April 2021.

The research activity is supervised by Prof. Giovanni F. Gronchi and it is focused on the modeling of complex dynamical systems by patching different dynamics.

## Research activity

We formalized the results obtained from the research carried out from November 2019 into a paper which was submitted to a scientific journal and which is currently under review.

We started a new research about the study of close approaches to the Earth. We developed a model inspired by Opik's theory ([R. Greenberg, 1988]) and its generalization developed by Valsecchi *et al.* (see [G.B. Valsecchi, 2003], [Valsecchi, 2006], [G.B. Valsecchi, 2015]). The model developed consists in patching of two Keplerian dynamics: when a celestial body, such as an asteroid, is sufficiently far from the Earth, we assume that it follows an unperturbed heliocentric orbit; on the contrary, when it is close to the planet and enters its sphere of influence, its unperturbed trajectory is considered planetocentric and hyperbolic. We take the Hill sphere as sphere of influence. We determined a set of canonical variables to study the planetocentric hyperbolic trajectories which are comparable to the so called Opik's variables. The Opik's variables consist in the hyperbolic excess velocity of the planetocentric orbit, two angles giving its orientation and two coordinates defined on the target plane, which is a plane intersecting the Earth center perpendicular to the incoming asymptote. Similarly, we introduced a set of canonical variables made by two coordinates defined on a plane which is parallel to the target plane, one coordinate defined along the incoming asymptote and their conjugated moments. We would like to use the model developed to study resonant close encounters. By considering the close encounter as instantaneous, it is possible to define a canonical transformation from an encounter to the successive one. Following [J.M. Maruskin, 2009], we would like to study the evolution of two-dimensional submanifolds parallel to some symplectic plane of the phase space (or admitting a symplectic parameterization) between two successive encounters.

In the period January-April 2021, I did an internship at the University of Padova under the supervision of Professor Christos Efthymiopoulos. The research topic was the motion of a small body (asteroid or comet) transiting from the exterior to the interior of the orbit of a planet (e.g. Jupiter). While the model is unique, various multipole truncations of the restricted three body problem lead to different approximations for the equations of motion inside or outside the orbit of Jupiter. In studying the problem, we arrived at the formulation of a closed-form normalization method which can be applied to approximate the Hamiltonian of a mass-less particle for which the tidal potential of the planet is an external perturbation. Currently, we are formalizing the work in a paper to be submitted for publication in a scientific journal.

## 1 Other Activities

Until December 2020, I attended the following courses:

- Complementi di Analisi Funzionale, by Professor A. Pratelli;

- Teoria e Metodi dell'Ottimizzazione, by Professor G. Bugi.

From March 15 to March 19, I virtually attended the Stardust-R Training School III, organized by the University Alexandru Ioan Cuza of Iasi (Romania) and the University of Roma Tor Vergata (Italy).

## References

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4/5/2021

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