Report on the Research Activity

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This report summarises the activities carried out by the research fellow Irene Cavallari during the six months of activity from May 2021 to October 2021.

The research activity is supervised by Prof. Giovanni F. Gronchi and it is focused on the modeling of complex dynamical systems by patching different dynamics.

Research activity

The research activity currently underway is about close encounters between the Earth and asteroids orbiting the Sun. We would like to apply the patched-conics approximation to model the problem [1]. This approximation consists in patching two Keplerian dynamics: when a celestial body, such as an asteroid, is sufficiently far from the Earth, we assume that it follows an unperturbed heliocentric orbit; on the contrary, when it is close to the planet and enters its sphere of influence, its unperturbed trajectory is considered planetocentric and hyperbolic. The application of the patched-conics method to study close encounters has not been deeply investigated. The main reason is the ambiguity about the extension of the sphere of influence (*soi*). Classically, the *soi* employed is either Hill's sphere or Laplace's sphere [2]. However, in [3] (where Laplace's sphere is employed) it emerges that the two-body approximation can fail depending on the relative velocity of the asteroid with respect to the planet. Also in [4] it is shown through a numerical study that the radius of the *soi* should depend on the relative velocity. Currently, we are studying close encounters with the goal to find a definition of *soi* alternative to the classical ones, based not only on the distance to the planet but also on the geometry of each particular encounter. For this purpose, we compare the trajectories computed by means of the patched-conics method using several values of the *soi*'s radius with the trajectory obtained through a numerical propagation of the restricted three-body problem. While applying this last model, we use the evolution of Tisserand's parameter to identify the end of the close encounter. Tisserand's parameter is defined as

$$T = \frac{1-\mu}{a} + 2\sqrt{(1-\mu)a(1-e^2)}\cos i,$$

with a, e, i the heliocentric semi-major axis, eccentricity and inclination respectively [1]. Even though close encounters can cause drastic changes of the heliocentric orbital elements of an asteroid, the value of Tisserand's parameter before and after a planetary encounter is almost the same. In particular, it drops during the encounter, but it approximately re-assumes the same value after it.

The interest in the patched-conics approximation applied to close encounters is in the framework of resonant returns. Indeed, it allows to treat resonant encounters by means of canonical transformations. We demonstrated that it is possible to construct a canonical transformation which corresponds to the map of the resonant return when we select initial conditions belonging to a subset of the phase space defined by the energy constraints. Canonical transformations have several advantages. By Poincaré theorem, the integral of Poincaré-Cartan is invariant (see [5]). Moreover, as explained in [6], it is possible to determine the evolution of a two-dimensional submanifold of the phase space which is parametrically symplectic (i.e. it admits a symplectic parametrization) using the State Transition Matrix.

In the last months, I continued to collaborate with Professor Christos Efthymiopoulos (University of Padova) on the research work started in January 2021. We examined a closed-form normalization method which can be applied to approximate the Hamiltonian of a mass-less particle for which the tidal potential of the planet is an external perturbation. We have formalized the work in a paper submitted for publication in a scientific journal.

1 Other Activities

- From September, I attend the course *Cosmologia del primo universo*, by Professor Dario Grasso.
- On October 15, I took the mid-Ph.D exam.
- From September 13 to September 17 , I virtually attended the Stardust-R Global Virtual Workshop II, organized by German Research Center for Artificial Intelligence (DFKI) and the University of Strathclyde (UK).
- From October 18 to October 22, I virtually attended the 364 IAU Symposium *Multi-scale (time and mass) dynamics of space objects.* During the event I presented the research work conducted in collaboration with Professor C. Efthymiopoulos.

References

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