Perturbed point-vortex: stability and twist dynamics

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Abstract

I will present our results about the stability and dynamics of a periodic Hamiltonian system in the plane with a singularity.

In a perfect fluid, a point-vortex is essentially a singularity of the vorticity, and can be modeled by the Hamiltonian $\Psi_0(x,y) = \frac{1}{2}\ln(x^2+y^2)$ being x and y the usual rectangular coordinates in the plane. The associated system is integrable, with the particles rotating around the vortex in circular paths and the origin is stable. If we introduce a external periodic perturbation $p(t,x,y)$ to this dynamical system, the corresponding Hamiltonian system is

$$\begin{cases} \dot{x} = \frac{y}{x^2+y^2} + \partial_y p(t,x,y) \\ \\ \dot{y} = -\frac{x}{x^2+y^2} - \partial_x p(t,x,y) \end{cases} \qquad (x,y) \in \mathcal{U} \setminus \{0\}$$

where $\mathcal{U}$ is a neighborhood of the origin. This system models ideally the passive advection (transport) of particles in a fluid subjected to the action of a steady vortex placed at the origin and a time-dependent background flow.

In this talk, I will focus mainly on the result concerning the stability, We will see which hypothesis must be imposed on the pertubation $p(t,x,y)$ to preserve the stability of the origin. In this context, the application of Moser's twist theorem allows us to find a family of invariant curves by the Poincaré map of our system. This is important since these curves surround the origin and act as barriers to the solutions; therefore, the stability of the origin will be guaranteed.

Concerning the dynamics, I will show which conditions on $p(t,x,y)$ are sufficient for the existence of periodic and generalized quasi-periodic trajectories.

This results are presented in the following works:

Marò, S.; Ortega, V.; Twist dynamics and Aubry-Mather sets around a periodically perturbed point-vortex. *(preprint)* , 2019.

Ortega, R.; Ortega, V.; Torres, P.; Point-vortex stability under the influence of an external periodic flow . *Nonlinearity* 31: 1849–1867, 2018.