Report on the Research Activity

Irene Cavallari

29/04/2022

This report summarises the activities carried out by the research fellow Irene Cavallari during the six months of activity from November 2021 to April 2022.

The research activity is supervised by Prof. Giovanni F. Gronchi and it is focused on the modeling of complex dynamical systems by patching different dynamics.

Research activity

The research activity currently underway is about the sphere of influence of the Earth. The concept of sphere of influence is useful in several contexts, such as the impact monitoring and the design of interplanetary trajectories for spacecrafts. In particular, it is a central element for the patched-conic method, which is a classic technique used to study planetary close-encounter [1]. However, this concept is still ambiguous, as different definitions exist in literature, typically based on the relative position between the planet and the body of interest [2]. The purpose of our research is to determine the most suitable definition of sphere of influence to be applied for the patched-conic method, such that the main features of a close encounter and of the post-encounter trajectory are well reproduced. We define the sphere of influence considering both the position and velocity of the body with respect to the planet. Our study relies on an optimization process, in which we compare the patched-conic orbit and the trajectory obtained in the framework of the circular restricted three-body problem: we minimize a suitable target function with respect to the radius of the sphere of influence. For the planar problem, we are collecting the results obtained in a database. We are also formalizing the work in a paper to submit for publication in a scientific journal.

In the period January-April 2022, I did an internship at the University of Tor Vergata under the supervision of Professor Giuseppe Pucacco. We have studied the bifurcation sequences of frozen orbits when the 2nd-order fundamental model of the satellite problem is augmented with further features of a typical planetary gravity field. The contribution of the octupolar term [3] and the relativistic correction due to the quadrupolar term [4] are considered. We have implemented a twice-reduced normal form [5, 6] and we have used a set of suitable variables [7], allowing to obtain in an efficient way the conditions for relative equilibria corresponding to the family of periodic orbits with fixed eccentricity and inclination. First of all, we have applied the method for the 2nd-order J_2-problem and we have succeeded in reproducing known results[8]. Then, we have applied the method to the above mentioned perturbations. Currently, we are formalizing the work in a paper.

Between January and March 2022, the following two papers were published:

- Cavallari I., Gronchi G.F., Baù G., (2022) *On the Sun-shadow dynamics*, Physica D: Nonlinear Phenomena;
- Cavallari I., Efthymiopoulos C., (2022) *Closed-form perturbation theory in the restricted three-body problem without relegation*, Celestial Mechanics and Dynamical Astronomy.

Other Activities

- From February 21 to February 23 , I virtually attended the Stardust-R Local Training Workshop II, organized by by the Delft University of Technology in collaboration with Hyperion Technologies BV.

References

[1] R. R. Bate, D. D. Mueller, and J. E. White. *Fundamentals of Astrodynamics.* Dover publications, 1971.

[2] G. A. Chebotarev. Gravitational Spheres of the Major Planets, Moon and Sun. *Soviet Astronomy*, 7:618, April 1964.

[3] Shannon L. Coffey, Andre Deprit, and Etienne Deprit. Frozen Orbits for Satellites Close to an Earth-Like Planet. *Celestial Mechanics and Dynamical Astronomy*, 59(1):37–72, May 1994.

[4] Joachim Heimberger, Michael Soffel, and Hanns Ruder. Relativistic effects in the motion of artificial satellites - The oblateness of the central body II. *Celestial Mechanics and Dynamical Astronomy*, 47(2):205–217, January 1990.

[5] G. Pucacco and A. Marchesiello. An energy-momentum map for the time-reversal symmetric 1:1 resonance with $\mathbb{Z}_2 \times \mathbb{Z}_2$ symmetry. *Physica D*, 271:10–18, 2014.

[6] Giuseppe Pucacco. Structure of the centre manifold of the L_1, L_2 collinear libration points in the restricted three-body problem. *Celestial Mechanics and Dynamical Astronomy*, 131:44, 2019.

[7] A. Marchesiello and G. Pucacco. Bifurcation sequences in the symmetric 1:1 hamiltonian resonance. *International Journal of Bifurcation and Chaos*, 26(04), 2015.

[8] J. F. Palacián. Dynamics of a satellite orbiting a planet with an inhomogeneous gravitational field. *Celestial Mechanics and Dynamical Astronomy*, 98(4):219–249, August 2007.

G I Gronchi