

## RELAZIONE SEMESTRALE 02/08/2022

ROBERTO CASTORRINI

Since February 2022 I am a member of the project *Regular and stochastic behaviour in dynamical systems: theory, applications, and interactions with other areas of Mathematics* supervised by Prof. Claudio Bonanno.

### 1. RESEARCH ACTIVITY

In the past six months (February 2022-July 2022) the following projects have been carried out:

#### Statistical properties of dynamical systems.

- *Quantitative statistical properties for partially hyperbolic systems.*  
I completed a joint work with Prof. Carlangelo Liverani ([1]). In this paper we study a class of two-dimensional partially hyperbolic systems, not necessarily skew products, in an attempt to develop a general theory. We provide explicit conditions for the existence of finitely many physical measures (and SRB) absolutely continuous with respect to Lebesgue with density in some Sobolev space, each mixing one having exponential decay of correlations. To illustrate the scope of the theory, we show that our results apply to the case of fast-slow partially hyperbolic systems.  
Recently the paper has been accepted on the international journal *Advances in Mathematics*.
- *Statistical properties of a class of two-dimensional triangle maps.*  
Jointly with Prof. Claudio Bonanno and Prof. Gianluigi Del Magno I am investigating the statistical properties of a two-dimensional dynamical systems generated by some *triangular maps* introduced in [4] in connection with a continued fraction algorithm. With the exception of [2], in which the authors studied a slow version of this kind of maps, nothing is known about the original triangle map from a quantitative statistical point of view, such as mixing and decay of the correlations. Besides being interesting from its connection with continued fraction algorithms, this maps are an example of a non-uniformly hyperbolic system in dimension greater than one, which is itself a challenging and relevant issue in dynamical systems.

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### Dynamical zeta functions.

- *Dynamical zeta functions for smooth parabolic maps.*

Jointly with Prof. Claudio Bonanno and Prof. Paolo Giulietti I am investigating the properties of dynamical zeta functions associated to general parabolic maps (e.g. expanding map with an indifferent fixed point) using the transfer operator approach. The attempt is to extend the results known only for a specific case, such as the Farey map ([3]), to obtain the germ of a general theory for this kind of maps which have a relevant interest, not only from a pure dynamical systems point of view, but also in physics and number theory.

- *Zeta functions for piecewise smooth expanding maps.*

We are investigating piecewise smooth uniform hyperbolic systems in any dimension using a functional approach. The aim is to study the spectrum of the transfer operator on a Banach space adapted to the discontinuous setting. We obtain a good bound on the essential spectral radius of the transfer operator using a thermodynamic formalism, which leads to a careful study of the dynamical zeta functions and dynamical determinants on which almost nothing is known about it in dimension higher than one.

This is a work in progress with Prof. Viviane Baladi. Most of the work has been essentially done and a first preliminary version of the paper will appear soon.

### Kinetic equations and mean-field approximation.

- *Mean field limit and propagation of chaos in the periodic Lorenz gas.*

Given a system of  $N$  particles moving in a dispersive billiard under the effect of an electric field and of a Gaussian thermostat, we derive a Vlasov-type PDE from the microscopic laws as the mean field limit of the number of particles. We also prove propagation of chaos. The novelty is the derivation of a Vlasov equation from a discontinuous microscopic dynamics.

Most of this work has been essentially done and the preprint will appear soon on arXiv. This is a joint work with Prof. Carlangelo Liverani.

## 2. TALKS, RESEARCH VISITS AND CONFERENCES

In the past six months (February 2022-July 2022) I participated in the following academic activities:

- February 17, 2022: Dynamics Seminar at Centro de Giorgi, Scuola Normale Superiore di Pisa (Invited talk).

**Title:** Quantitative statistical properties for a class of partially hyperbolic systems.

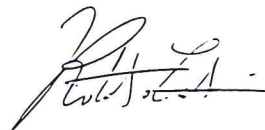
**Abstract:** In the last few years, an extremely powerful method has been developed to study the statistical properties of a dynamical system: the functional approach. It consists of the study of the spectral properties of transfer operators on suitable Banach spaces. In this talk I will discuss how to further such a point of view to a class of two dimensional partially hyperbolic systems, not necessarily skew products, in order to provide explicit conditions for the existence of finitely many physical measures and prove exponential decay of correlations for mixing measures. To illustrate the scopes of the theory, I will discuss how to apply the

results to a family of fast-slow partially hyperbolic maps. This is a joint work with Carlangelo Liverani.

- March 27- April 01, 2022: Visiting Prof. Viviane Baladi at LPSM (Sorbonne Université, Paris) to continue working on the project *Zeta functions for piecewise smooth expanding maps*. The visit and the research was supported by the European Research Council (ERC) under the European Unions Horizon 2020 research and innovation program (grant agreement No 787304).
- June 06-June 09, 2022: Dinamici VII: Riemann International school of mathematics (Varese).

#### REFERENCES

- [1] R. Castorri, C. Liverani. *Quantitative statistical properties for two dimensional partially hyperbolic systems*. Preprint: <https://arxiv.org/abs/2007.05602>, (2021).
- [2] C. Bonanno, A. Del Vigna, S. Munday, *A slow triangle map with a segment of indifferent fixed points and a complete tree of rational pairs*, Monatshefte für Mathematik, 194 (2021), 1–40.
- [3] C. Bonanno, S. Isola, *A thermodynamic approach to two-variable Ruelle and Selberg zeta functions via the Farey map*. Nonlinearity 27 (2014), 897926
- [4] T. Garrity.: *On periodic sequences for algebraic numbers*. J.NumberTheory88(1),86103(2001)



Per presa visione.  
Pisa, 02/08/2022



Prof. Claudio Bonanno

